TURNER

Investigation of an empirical formula for the calculation of self-inductance

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INVESTIGATION OF AN EMPIRICAL FORMULA FOR THE CALCULATION OF SELF-INDUCTANCE

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

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INTRODUCTION.

The practicing engineer is confronted many times with the prolem of designing air cored coils of given inductance and current carrying. Now although this problem presents no serious difficulty to the technical man it is necessary for him,unless he frequently makes such calculations,to find out what particular formula will give the desired approximation,and to spend much time speculating upon the design.

The Bureau of Standards has issued many bulletins upon the calculation of self-inductance. These,although most useful to the physicist and mathematician,are are of little value ,however, on account of their mathematical nature,to the average power plant engineer who lacks the technical education necessary to understand and make use of them. Moreover,while practically all formulae which have been proposed are extremely accurate for coils of given proportions they fail to meet the requirements of the practicing engineer because their errors may increase markedly with a decided change of the proportions of the coil.

It is the object of this paper to investigate an empirical formula,proposed by Professor Brooks of the University of Illinois, for determining with close approximation the inductance of a coil of wire wound cylindrically. This formula is correct within two or three per cent for almost any shaped coil and since by the method employed by the Bureau of Standards it is difficult to measure the dimensions of a coil within one or two per cent,depending upon the size of the coil,the result obtained by Professor

Brooks' formula are as correct as the usual data. It will be observed that the inductance depends not at all upon the size of wire used except in so far as it controls the number of turns that it is possible to wind within the given space,nor does it depend upon the method of winding. The latter will not be investigated in this paper but experiments in general indicate that regularity of winding has no influence upon the inductance,except as to the space occupied by the wire;that a coil of wire irregularly wound within a given space and filling it completely gives the same inductance as the same wire wound regularly,but with heavier insulation to fill up the same amount of space,the insulation being uniform between the layers.

The value of this formula will be proven first by comparing it with formulas known to be accurate for special cases,second by checking the value found by its use with results obtained by electrical measurements,and third,by comparing the value of inductance as determined by this formula with the known values of inductanceof coils of various sizes,shapes,and number of turns.

The variation of inductance of reactance coils with the mean radius,axial length of coil,gauge number,and number of turns the total length of conductor remaining constant,and with the length of conductor the mean radius or the axial length of coil remaining constant will be discussed with the hope that the discussion may be of use to one designing such coils.

PROFESSOR BROOKS' FORMULA.

$$L = \frac{C^2}{1+t+.5D} \times F' \times F'' \text{ where}$$

$$F' = \frac{10\,1 + 12\,t + D}{10\,1 + 10\,t + 1.4D} \quad \text{and}$$

$$F'' = .5 \log\left(100 + \frac{7D}{2\,1 + 3\,t}\right) \text{ where}$$

$C = 2\,\pi\,a\,n$,

a = mean radius,

n = total number of turns of wire in coil,

l (letter) = axial length of coil includind the insulation of first and last wire,

t = thickness of coil,

D = outside diameter,

$D - t = 2\,a$ = meandiameter.

All dimensions are in centimeters and the value of inductance is in absolute units or centimeters. To reduce centimeters to the practical units or henries divide by 10^9.

$\frac{C^2}{1 + t + .5\,D}$ = T' will be called the first term and F' and F" will be known as the first and second factors respectively.

COMPARISON WITH FORMULA FOR LONG SOLENOIDS.

For long solenoids F' and F" reduce to unity since t and D in the first factor are negligible in comparison with 1 and 10 1 over

10 l equals unity and in the same manner 7 D/(2 l + 3 t) in the second factor may be neglected for large value of l and F" reduces to .5 log of 100 which is equal to unity and in like manner (t +.5D) in the first term may also be neglected in comparison with l and the formula reduces to

$$L = C^2/l = (2\ \pi\ a\ n)^2/l$$

which is the formula given in most text books for long solenoids.

COMPARISON WITH KIRCHHOFF'S FORMULA FOR CIRCULAR RINGS.

For circular rings the length and the thickness are negligible in comparison with D and the first term reduces to $C^2/.5D$ which equals $2\ C^2/D$, multiplying both numerator and denominator by π, $T' = 2\ \pi\ C^2/\pi\ D = 2\ \pi\ C = 6.286\ C$, in the same way F' reduces to D/.7D = 1.43. In F" l and t cannot be neglected since they enter as multiplying factors, but, since l and t are equal and 100 may be neglected when D is large, F" reduces to .5 log 7D/5D = .5 log 1.4D/t multiplying both numerator and denominator by π, F" = .5 log $1.4\pi D/\pi t$ but since π D = C we have F" = .5 log 1.4 C/π t = .5 log .7 C/π r where r is the radius of the wire. Substituting these values in the equation for L, we have

$$L = 6.28\ C \times 1.43 \times .5 \log .7\ C/\pi\ t$$

$$= 4.6\ C \log .7\ C/\pi\ r$$

$$= 4.\overset{49}{6}\ C \log .223\ C/r$$

$$L = 2\ C(\log_e C/r\ -1.508) \quad \text{Kirchhoff's Formula.}$$

$$= 2\ C\ (2.3 \log_{10} C/r\ - 1.508)$$

$$= 2\ C\ (2.3 \log C/r - 2.3 \times .65)$$

$$= 4.6\ C\ (\log C/r - .65)$$

Now let $\log x = .65$, or

$$x = 4.5$$

Substituting log x for .65 and we get

$$L = 4.6\ C\ (\log C/r - \log x)$$

$$= 4.6\ C \log C/r\ x$$

Substituting the value found for x the equation reduces to

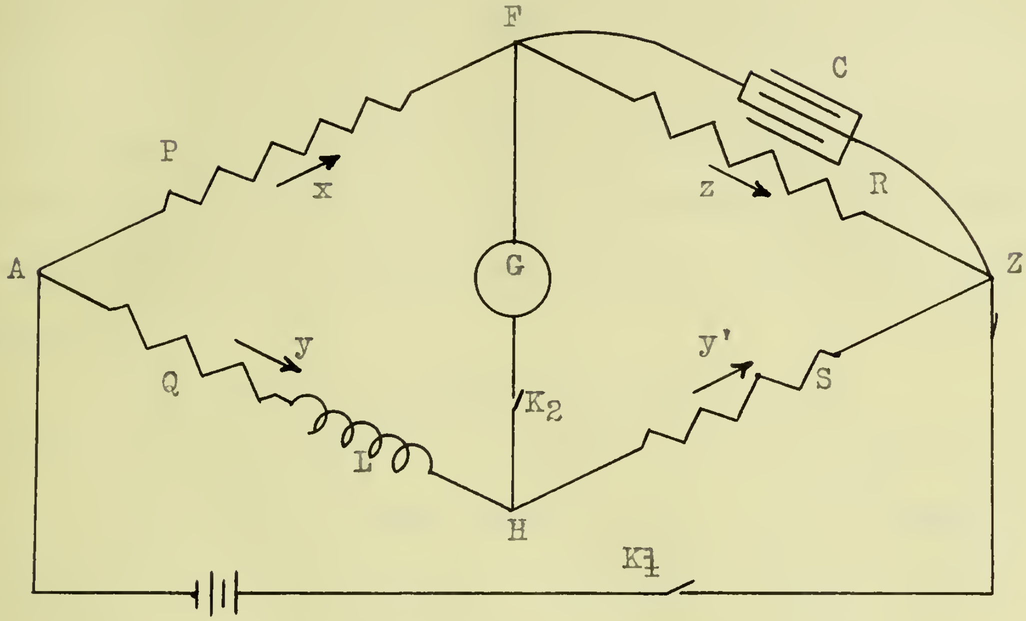
$$L = 4.6\ C \log C/4.5\ r$$

$$= 4.6\ C \log .221\ C/r$$

Therefore these two equations reduce to the same form.

Following this will be given, for convenience of reference, the development of the different methods of measuring electrically the self-inductance of coils which were used in checking this formula.

COMPARISON OF THE SELF-INDUCTANCE OF A COIL WITH THE CAPACITY OF A CONDENSER. MAXWELL.



When the battery circuit is closed, the potential difference at the terminals of R causes a current to flow through it and at the same time charges the condenser C. The potential difference rises as the condenser receives its charge, and therefore the current through R requires a definite time-interval to reach its final value.

The current through the coil Q will increase from zero to its maximum in a precisely similar way on account of the counter electromotive force of self-inductance. Both the condenser and the coil have a time constant; and the effect of the condenser in delaying the current in one branch may be made to offset that of the coil in the other, so that the rise of potential at F may be the same as that at H. In that case no current will pass through the galvanometer. We have to determine the conditions under which the potential at F remains equal at every instant to that at H.

Let x and z be the quantities which have passed through P and R

respectively at the end of the interval t after closing the circuit. Then $x-z$ will be the charge of the condenser at the same instant.

The potential difference between the two sides of the condenser is by Ohm's law $R\ dz/dt$, since dz/dt is the value of the current.

Therefore $\quad x-z = RC\ dz/dt \qquad (1)$

Let y be the quantity traversing Q in the same time t. Then the potential difference between A and H is equal to that between A and F when there is a balance and no current flows through the galvanometer; or

$$Q\ dy/dt + L\ d^2y/dt^2 = P\ dz/dt \qquad (2)$$

The first member consists of the effective electromotive force producing a current and an electromotive force of self-induction. The sum of the two is the potential difference between A and H.

Since there is no current through the galvanometer the quantity passing along HZ must be the same as that along AH, or $y' = y$.

Therefore $\quad S\ dy/dt = R\ dz/dt, \qquad (3)$

since the potential difference between F and Z is the same as that between H and Z, when no current flows through the galvanometer.

From (1) $\quad dx/dt - dz/dt = RC\ d^2z/dt^2,$

the rate at which the condenser is charged. Substitute in (2) and

$$Q\ dy/dt + L\ d^2y/dt^2 = P(RC\ d^2z/dt^2 + dz/dt)$$

From (3) $\quad dy/dt = (R/S)\ dz/dt$. Substituting in the last equation

$$(QR/S)\ dz/dt + (LR/S)\ d^2z/dt^2 = P(RC\ d^2z/dt^2 + dz/dt).$$

Multiply by S and integrate,

$$QRz + LR\ dz/dt = PRSC\ dz/dt + PSz, \text{ or}$$

$$QR(1 + (L/Q)\ d/dt)z = PS(1 + (RC)\ d/dt)z \qquad (4)$$

This is the equation of condition that no current shall pass through the galvanometer.

The condition for a steady current with a Wheatstone's bridge is

$$QR = PS \tag{5}$$

Hence the condition that no current shall traverse the galvanometer when the battery is opened and closed is,

$$L/Q = RC \tag{6}$$

L/Q and RC are called the time constants of the coil and the condenser respectively. If by varying P and R the bridge can be adjusted so that no current traverses the galvanometer on opening and closing the battery circuit, as well as when it is kept closed, then the two "time constants" are equal and

$$L = QRC.$$

To show that a time constant is a time, since resistance has the dimensions of a velocity, and a capacity is the square of a time divided by a length, we have from the equation $L'/Q = RC$ (calling the coefficient of self-induction L' to distinguish it from a length L)

$$L'/(L/T) = (L/T)(T^2/L) = T$$

also $$L' = (L/T)^2(T^2/L) = L,$$

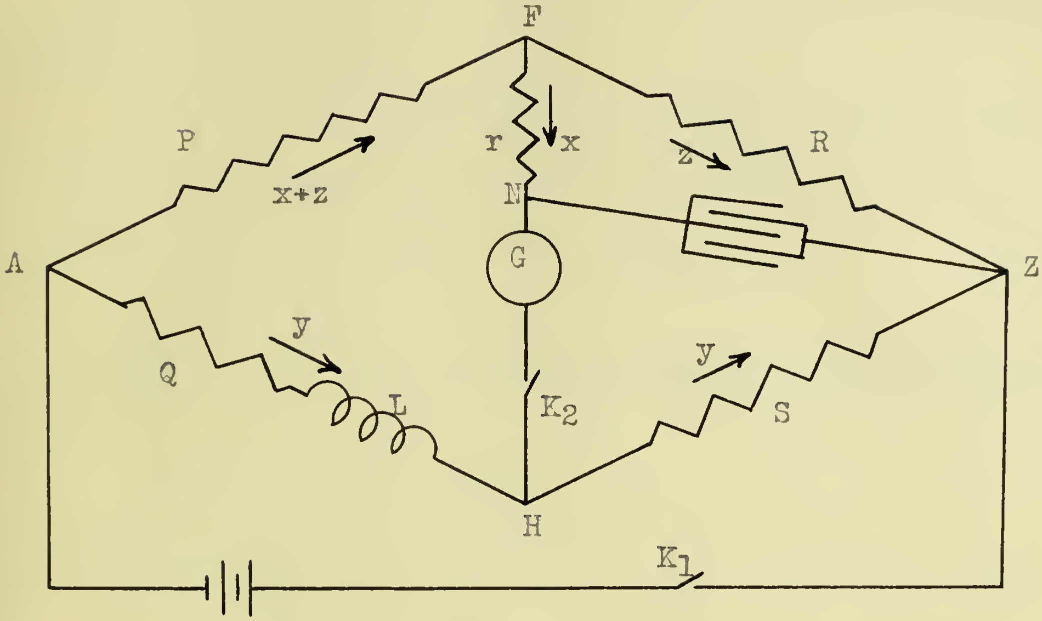
or self-induction is a length. The unit of self-induction is the henry and equals 10^9 centimeters. It varies directly as the ohm.

If C is in microfarads the value of L from the equation above will be a million times too large and must be multiplied by 10^{-6} to reduce to henries.

ANDERSON'S MODIFICATION OF MAXWELL'S METHOD.

In the preceding method of Maxwell a double adjustment must be made in order to effect a balance. First, one of the branches P has to be adjusted for a balance with steady currents. Then, in order to obtain a balance when the galvanometer circuit is closed first, the

resistance R will have to be adjusted.



This necessitates a new adjustment of P, and so on. Anderson's modification of Maxwell's method is designed to facilitate the adjustment.

Suppose a balance has been obtained for steady currents by closing K_1 before K_2. This balance will not be disturbed by introducing the resistance r between F and N. Adjust t therefore till the galvanometer shows no deflection when K_2 is closed before K_1. The potential at H and N then remain equal to each other. Let x be the quantity which has flowed into the condenser at the time t, and z be the quantity which has passed through FZ. Then, x+z has passed through AF. Then if C is the capacity of the condenser, since the fall of potential from F to Z is the same by the two paths, we have

$$R\, dz/dt = (x/C) + r\, dx/dt \qquad (1)$$

Also since N and H must be of the same potential,

$$x/C = S\, dy/dt \qquad (2)$$

Further, the change of the potential from A through F to N is the

same as from A to H. Hence

$$r\,dx/dt + P(dx/dt + dz/dt) = Q\,dy/dt + L\,d^2y/dt^2. \quad (3)$$

Substituting from (1) and (2),

$$(r+P)dx/dt + P/R(x/C + r\,dx/dt) = Qx/SC + L/SC\,dx/dt.$$

This equation expresses both conditions necessary for a balance with variable currents. For steady currents

$$P/R = Q/S.$$

Hence the other condition is found by equating the coefficients of dx/dt; or

$$r + P + Pr/R = L/SC.$$

This condition gives the formula

$$L = C(r(Q+S) + PS).$$

If r is zero, $L = CPS = CQR$, which is Maxwell's formula.

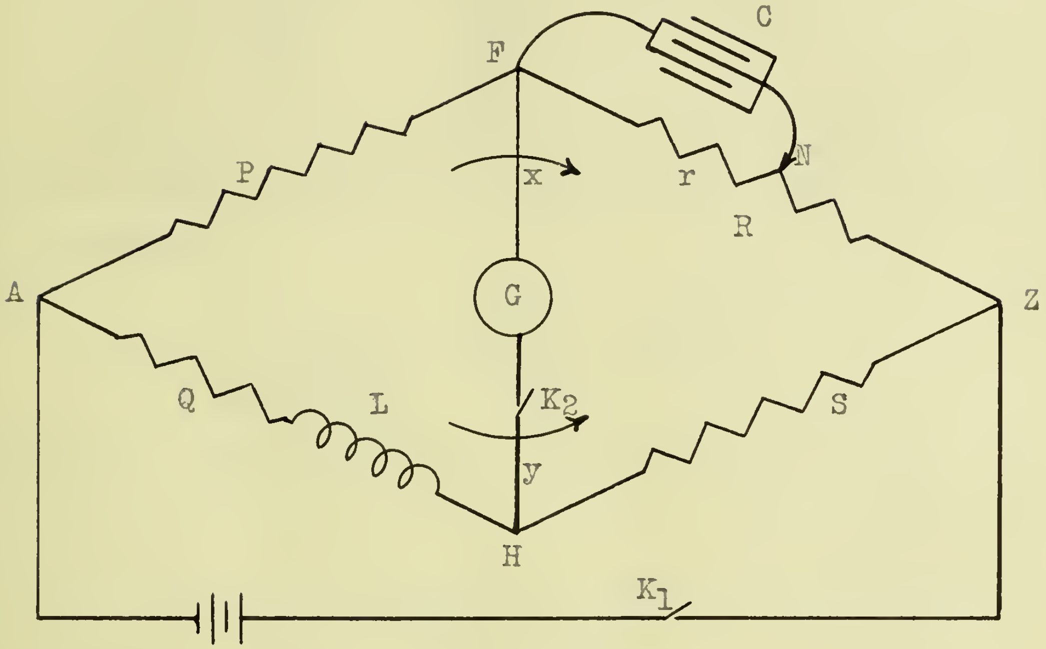
To apply the above equation for L, first obtain a balance in the ordinary way, and then adjust r and, if possible, C till there is no deflection of the galvanometer needle on working K_1 with K_2 closed.

For sensitiveness of the finel balance it is desirable to make R and S large, and r small. Since Q is usually small, P will also be small.

RIMINGTON'S MODIFICATION OF MAXWELL'S METHOD.

In this method one side of the condenser is connected to F, and the other to the point N, which can be shifted along so as to vary r without any change in the resistance of R of that branch. In this arrangement the discharge through the galvanometer, due to the discharge of the condenser and the self-induction of the coil, are in opposite directions and equal, when balances have been secured. Let y be the current flowing in the arms Q and S, when it has reached its

RIMINGTON'S MODIFICATION OF MAXWELL'S METHOD.



Although the primary object of this paper is not a study of the methods of measuring self-inductance it was observed that Rimington's modification of Maxwell's method gave the most consistant results and required less time to obtain a balance.

steady value, and x that in P and R.

Let both keys be closed and then let K_1 be opened. The quantity of electricity which passes through the galvanometer, due to the self-induction in Q, is

$$\frac{Ly}{P+Q+\frac{G(R+S)}{G+R+S}} \times \frac{R+S}{G+R+S} = \frac{Lya}{P+Q+Ga}.$$

This is the integral of the current between the limits 0 and y. The quantity passing through the galvanometer from the discharge of the condenser is

$$Cxr\frac{r}{P+Q+\frac{G(P+Q)}{G+P+Q}} \times \frac{P+Q}{G+P+Q} = \frac{Cxr^2b}{R+S+Gb}.$$

This discharge passes while the current through r falls from x to 0.

These quantities pass through the galvanometer in opposite directions and if there is no deflection,

$$\frac{Lya}{P+Q+Ga} = \frac{Cxr^2b}{R+S+Gb}.$$

But $\quad \dfrac{Lya}{P+Q+Ga} = \dfrac{Ly(R+S)}{c}$

and $\quad \dfrac{Cxr^2b}{R+S+Gb} = \dfrac{Cxr^2(P+Q)}{c}$

Hence $\quad Ly(R+S) = Cxr^2(P+Q)$

And $\quad L = Cr^2(x/y)(P+Q)/(R+S)$

Now $\quad x/y = Q/P$

Therefore $\quad (x/y)(P+Q)/(R+S) = (Q/P)(P+Q)/(R+S) = Q/R,$

since $\quad PS = QR.$

Hence $\quad L = Cr^2Q/R.$

If r = R, we have Maxwell's formula,

$$L = CQR.$$

The resistance must be such that r can be adjusted without changing the value of R after a balance has been obtained for steady currents.

The condition $L = Cr^2Q/R$ may be obtained directly from Maxwell's equation $L = CQR$. When no deflection of the galvanometer is observed on opening the battery circuit,a certain quantity of electricity,coming from the condenser,must pass through the branch S. If one terminal is moved along R to the point N,the fraction of the charge passing through S will be decreased in the ratio of r/R;and as the total charge will be decreased in the same ratio because of the lower potential to which the condenser is charge, the quantity passing through S on discharge will be reduced in the ratio r^2/R^2. Consequently,if the same quantity is to pass through S as in Maxwell's method,the capacity of the condenser must be increased in the ratio of R^2/r^2. Whence it follows that

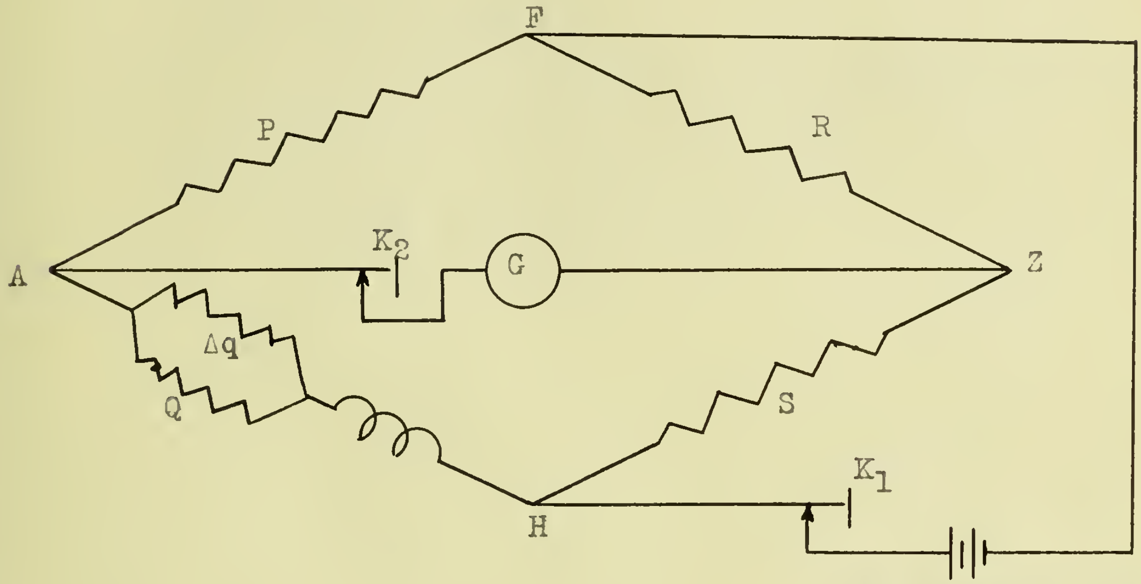
$$L = CQr^2/R.$$

RUSSELL'S MODIFICATION OF MAXWELL'S METHOD.

Connect the coil exactly as in Maxwell's method and balance for steady currents. Then if the galvanometer key be closed first,there will be a throw of the needle when the battery key is closed;and if the battery key be opened first the throw of the needle will be in the other direction. Now connect the condenser,which should be a subdivided one,as a shunt to the branch R. The effect will be to reduce the throw of the needle. Using different values of the condenser capacity,one giving a throw in one direction on opening or closing the battery circuit,and the other a throw in the other

direction. Then by interpolation find the capacity which would reduce the deflection to zero. This capacity, substituted in the equation $L = QRC$, gives the desired inductance.

SELF-INDUCTION. LORD RAYLEIGH'S.



Two Storage Cells.

$$P : R = Q : S$$

$g = ke$, where g is the current through the galvanometer,

$$e = -\,d(li)/dt = -L\,di/dt,$$

$$g = -Lk\,di/dt,$$

But the quantity flowing by a point $= Q = It$, then

$$dQ = g\,dt = -kL\,di/dt = -kL\,di,$$

$$Q = -kL\int_I^0 di = kLI.$$

For a ballistic galvanometer,

$$Q = (HT/2\pi G)\ 2 \sin \alpha/2\ (1+\lambda/2)$$

H = strength of in which the coil swings,

T = time of a complete peroid,

G = is a constant depending upon the dimensions of coil.

= angular throw of needle.

$kLI = (HT/\pi G) \sin \alpha/2 \ (1+\lambda/2)$

$L = (HT/\pi GkI) \sin \alpha/2 \ (1+\lambda/2)$

$g' = ke'$ for steady currents,

$g' = k\Delta qI$ also

$g' = (H/G) \tan \theta$

$(H/G) \tan \theta = k\Delta qI$,

$kI = (H/G\Delta q) \tan \theta$ which may be substituted for kI,

$L = (HT\Delta qG/\pi GH \tan \theta) \sin \alpha/2 \ (1+\lambda/2)$

$L = (T\Delta q/\pi)(\sin \alpha/2(1+\lambda/2))/\tan \theta$

RIMINGTON'S METHOD.

Coil # 1.

C	r	Q	Q'	R_L	a	P	S	R	L
M.F.	Ohms	Ohms	Ohms	Ohms	mm.	Ohms	Ohms	Ohms	Mil Henries
6.89	392	29.330	19.7	8.99	844	30	400.100	410	75.498
6.89	393	29.223	19.7	8.99	833	30	400.107	410	75.856
6.39	408	29.223	19.7	8.99	833	30	400/107	410	75.816
6.89	393	29.223	19.7	8.99	833	30	400.107	410	75.856
6.89	437	29.355	19.8	8.99	883	30	500.075	510	75.721
6.39	454	29.353	19.8	8.99	879	30	500.080	510	75.804
5.89	472	29.350	19.8	8.99	875	30	500.080	510	75.514
5.99	469	29.350	19.8	8.99	875	30	500.080	510	75.825
6.89	393	29.221	19.8	8.99	674	30	400.201	410	75.842
6.39	408	29.221	19.8	8.99	674	30	400.209	410	75.810
								Av.	75.754

Coil # 4.

C	r	Q	Q'	R_L	a	P	S	R	L
5.00	192	30.051	23.01	6.76	597	30	300.258	300	18.52
6.39	201	29.346	22.20	6.76	572	30	400.254	410	18.47
3.54	270	29.434	22.20	6.76	726	30	400.166	410	18.52
3.04	291	29.425	22.20	6.76	727	30	400.175	410	18.51
3.04	290	29.433	22.20	6.76	739	30	400.167	410	18.36
								Av.	18.476

Coil # 5.

C	r	Q	Q'	R_L	a	P	S	R	L
6.09	320	19.660	11.5	7.83	516	20	500.310	510	24.04
4.04	320	29.734	21.2	7.83	631	30	500.236	510	24.11
3.14	364	29.728	21.2	7.83	622	30	500.242	510	24.25
5.89	268	29.183	21.2	7.83	551	30	500.287	510	24.20
6.39	256	29.182	21.2	7.83	549	30	500.281	510	23.96
								Av.	24.11

ANDERSON'S METHOD.

Coil # 2.

C	r	Q	Q'	R_L	a	P	S	R	L
M.F.	Ohms	Ohms	Ohms	Ohms	mm.	Ohms	Ohms	Ohms	Mil Henries.
0.2	190.0	29.985	26.1	3.48	633	20	100.239	100	5.550
0.7	50.0	19.982	16.1	3.48	628	20	100.238	100	5.610
1.0	78.6	20.009	16.1	3.48	651	20	40.200	40	5.537
0.7	49.0	19.987	16.1	3.48	636	20	100.233	100	5.527
0.1	140.0	19.987	16.1	3.48	636	20	100.233	100	5.490
								Av.	5.543

Coil # 3.

C	r	Q	Q'	R_L	a	P	S	R	L
1.00	110.0	40.019	34.7	4.90	655	40	40.221	40	10.435
0.40	200.0	19.990	14.7	4.90	617	20	100.245	100	10.420
1.00	57.0	30.010	24.7	4.90	641	30	100.230	100	10.429
0.70	39.0	29.987	24.7	4.90	605	30	100.230	100	10.490
0.40	420.0	19.996	14.6	4.90	775	20	40.144	40	10.424
								Av.	10.439

Coil # 4.

C	r	Q	Q'	R_L	a	P	S	R	L
2.85	74.0	30.157	23.0	6.76	612	30	40.248	40	18.30
2.85	26.4	30.134	23.0	6.76	584	30	100.266	100	18.40
2.85	37.4	20.139	13.0	6.76	592	20	100.261	100	18.50
0.50	289.4	20.139	13.0	6.76	592	20	100.261	100	18.40
0.50	136.0	30.072	23.0	6.76	472	30	200.328	200	18.60
								Av.	18.44

Coil # 5.

C	r	Q	Q'	R_L	a	P	S	R	L
M.F.	Ohms	Ohms	Ohms	Ohms	mm.	Ohms	Ohms	Ohms	Mil Henries.
2.85	2.4	39.99	32.00	7.83	251	40	200.48	200	24.197
1.00	79.0	29.96	21.70	7.83	674	30	200.21	200	24.189
0.40	440.0	29.94	21.70	7.83	648	30	100.23	100	24.072
2.85	42.0	29.94	21.70	7.83	648	30	100.23	100	24.150
2.85	10.7	29.92	21.70	7.83	614	30	200.25	200	24.144
								Av.	24.126

LORD RAYLEIGH'S METHOD.

Use one storage cell.

Always make delta r negative.

Coil # 4.

d_α	d_θ	α	$\sin \alpha/2$	θ	$\tan \theta$	Δr	R=P	$Q \mp S$,	L
57.0	79.0	3°12'	.0279	4°25.3'	.0773	.0280	80	26	18.50
61.0	84.5	3°25.3'	.0299	4°43.7'	.0827	.0280	70	26	18.50
65.0	79.0	3°38.7'	.0318	4°25'	.0773	.0245	90	16	18.40
74.5	91.0	4°11'	.0365	5°05'	.0890	.0245	40	16	18.40
62.0	75.6	3°28.7'	.0304	4°14'	.0740	.0245	60	16	18.40
								Av.	18.444

RUSSELL'S METHOD.

Coil # 4.

			C	P	Q'	Q	R	S'	a	L
C	1.00	12.32	9.249	20	13	20.05	100	100	630	18.50
d	+51.00	-19.00								
C	3.05	11.32	9.253	20	13	20.05	100	100	631	18.60
d	+39.00	-13.00								
C	1.00	5.90	2.818	20	13	20.01	300	300	684	18.50
d	+17.00	-23.00								
C	.80	4.05	2.646	20	13	20.01	350	350	684	18.50
d	+15.80	-12.00								
									Av.	18.552

Coil # 5.

			C	P	Q'	Q	R	S'	a	L
C	.00	.80	2.667	30	22.1	30.04	300	300	323	24.00
d	+10.00	+7.00								
C	.00	1.00	2.658	30	22.1	30.04	300	300	335	24.00
d	+10.00	+6.00								
C	.00	6.89	3.244	30	22.1	30.05	250	250	352	24.40
d	+11.00	-13.00								
C	.00	6.89	3.191	30	22.1	30.05	250	250	352	24.00
d	+11.00	-13.50								
									Av.	24.10

IMPEDANCE METHOD OF MEASURING THE COEFFICIENT OF SELF-INDUCTION.

The value of the coefficient of self-induction of a coil of known resistance R may be found by passing through it an alternating current and measuring the potential difference between its terminals by means of a voltmeter. At the same time the current through the coil should also be measured. Then

$$I^2 = E^2/(R^2+L^2\omega^2,$$

where E is the measured potential difference, I the current, R the ohmic resistance of the coil, and L the inductance in henries. In this case it is necessary to know the frequency of the current which may be obtained from the speed of the generator. This might be called the engineering method of determining the self-inductance and while ordinarily it will not give the degree of accuracy that the methods previously described will but for most work it gives sufficiently accurate results.

DATA.

Coil # 20.

E	I	Z	R	f	Mil Henries.
2.62	345	.00760	.00454	142	.00686
3.09	403	.00767	.00447	143	.00690
2.41	324	.00746	.00447	141	.00670
		Av.	.00450	Av.	.00684

Coil # 21.

E	I	Z	R	f	Mil Henries.
4.61	219	.0211	.00768	135	.0230
4.26	202	.0211	.00776	135	.0230
5.65	265	.0213	.00777	134	.0236
6.41	311	.0206	.00760	133	.0229
7.00	341	.0205	.00760	131	.0231
5.56	268	.0208	.00768	132	.0231
		Av.	.00768	Av.	.0231

Coil # 22.

E	I	Z	R	f	Mil Henries.
5.79	150	.0386	.00893	134	.0446
7.10	185	.0384	.00907	133	.0447
8.78	225	.0390	.00907	135	.0447
7.58	194	.0391	.00913	135	.0448
		Av.	.00905	Av.	.0447

DATA.

Coil # 23.

E	I	Z	R	f	Mil Henries.
7.40	147	.0500	.0145	114	.0665
8.32	165	.0504	.0144	113	.0670
8.38	167	.0501	.0145	114	.0665
9.08	182	.0500	.0145	112	.0667
9.61	195	.0493	.0143	112	.0667
10.14	205	.0494	.0144	111	.0677
10.52	214	.0492	.0146	111	.0677
9.73	197	.0494	.0145	112	.0667
9.22	184	.0502	.0145	114	.0665
		Av.	.0145	Av.	.0669

Coil # 24.

E	I	Z	R	f	Mil Henries.
12.04	106	.1131	.0279	118	.147
12.05	110	.1103	.0278	116	.146
12.78	113	.1132	.0285	120	.145
12.30	109	.1131	.0285	119	.146
12.58	112	.1124	.0283	118	.146
12.85	114	.1130	.0278	119	.146
13.08	117	.1121	.0278	117	.147
12.75	113	.1130	.0283	119	.146
12.45	111	.1122	.0281	118	.146
				Av.	.146

DATA.

Coil # 25.

E	I	Z	R	f	Mil Henries.
2.15	304	.00707	.00408	119	.00865
2.35	326	.00721	.00446	119	.00776
2.25	313	.00721	.00433	118	.00776
2.32	316	.00734	.00434	118	.00792
2.26	309	.00731	.00422	119	.00794
2.40	319	.00752	.00428	119	.00826
2.45	329	.00745	.00423	119	.00815
2.42	326	.07744	.00433	119	.00812
2.32	313	.00741	.00427	119	.00809
2.22	296	.00755	.00429	119	.00824
			Av. .00428	Av.	.00809

Coil # 26.

E	I	Z	R	f	Mil Henries.
4.24	234	.0181	.00854	118	.0217
4.40	247	.0178	.00838	118	.0212
4.58	253	.0181	.00830	118	.0217
4.72	263	.0179	.00836	117	.0213
4.81	267	.0180	.00850	118	.0214
5.01	280	.0179	.00840	118	.0214
5.21	290	.0179	.00838	118	.0213
5.01	280	.0179	.00844	118	.0213
4.80	267	.0179	.00840	118	.0214
4.73	263	.0180	.00842	118	.0213
					Av. .0214

DATA.

Coil # 27.

E	I	Z	R	f	Mil Henries.
				f	
4.56	96	.0476	.0160	121	.0591
4.92	103	.0477	.0160	121	.0591
5.10	107	.0476	.0158	121	.0591
5.43	114	.0476	.0158	121	.0591
5.80	121	.0480	.0160	121	.0595
6.10	128	.0477	.0161	121	.0591
6.36	134	.0474	.0159	121	.0587
6.20	132	.0470	.0159	121	.0582
5.94	125	.0475	.0160	121	.0588
5.6	118	.0475	.0158	121	.0588
				Av.	.0589

Coil # 28.

6.70	67	.100	.0296	119	.127
7.25	73	.099	.0296	118	.127
8.38	83	.101	.0293	120	.128
9.20	91	.101	.0293	118	.128
9.40	94	.100	.0299	120	.127
9.82	98	.100	.0295	119	.127
9.48	94	.101	.0296	119	.128
9.15	91	.101	.0295	119	.128
8.58	85	.101	.0295	119	.128
8.00	79	.101	.0296	119	.128
				Av.	.128

DATA.

Coil # 29.

E	I	Z	R	f	Mil Henries.
2.12	303	.00698	.00496	119	.00667
2.20	310	.00708	.00485	119	.00682
2.22	318	.00698	.00494	119	.00667
2.37	332	.00714	.00480	119	.00694
2.47	342	.00722	.00487	119	.00704
2.49	345	.00722	.00488	119	.00704
2.66	359	.00740	.00490	119	.00744
2.62	361	.00726	.00489	119	.00716
2.74	370	.00740	.00488	119	.00744
2.81	379	.00742	.00489	119	.00744
				Av.	.00716

Coil # 30.

E	I	Z	R	f	Mil Henries.
3.41	200	.0170	.00852	118	.0203
3.62	214	.0169	.00840	118	.0200
3.90	231	.0169	.00831	118	.0200
4.17	247	.0168	.00820	118	.01990
4.35	257	.0169	.00832	118	.0200
4.48	263	.0170	.00835	118	.0203
4.68	276	.0170	.00835	118	.0203
5.04	295	.0171	.00844	118	.0203
4.81	282	.0171	.00843	118	.0203
4.78	280	.0171	.00845	118	.0203
				Av.	.0202

DATA.

Coil # 31.

E	I	Z	R	f	Mil Henries.
6.02	137	.0438	.0161	123	.0540
6.92	156	.0443	.0146	120	.0546
7.10	163	.0436	.0144	122	.0534
6.95	159	.0437	.0146	122	.0538
6.68	152	.0440	.0152	120	.0542
6.24	143	.0437	.0150	120	.0537
5.67	132	.0430	.0151	121	.0528
5.37	122	.0440	.0152	120	.0542
4.52	105	.0430	.0151	121	.0528
6.02	140	.0430	.0151	121	.0528
				Av.	.0536

Coil # 32.

6.90	77.0	.0896	.0288	116	.116
7.30	82.2	.0888	.0282	116	.115
8.80	96.8	.0908	.0281	116	.118
9.65	107.2	.0889	.0281	116	.116
10.28	114.7	.0896	.0285	116	.116
10.90	122.5	.0884	.0284	116	.115
11.13	125.8	.0887	.0285	116	.115
10.48	117.2	.0894	.0285	116	.116
9.66	107.0	.0902	.0282	116	.117
8.92	98.0	.0910	.0280	116	.118
				Av.	.116

DATA.

Coil # 33.

E	I	Z	R	f	Mil Henries.
2.14	329	.0065	.00434	118	.00660
2.10	318	.0066	.00424	118	.00673
2.26	342	.00662	.00424	118	.00687
2.40	359	.00668	.00428	118	.00698
2.45	361	.00678	.00412	118	.00712
2.54	374	.00668	.00412	118	.00698
2.47	361	.00684	.00423	118	.00724
2.32	342	.00678	.00423	118	.00712
2.14	318	.00674	.00422	118	.00698
2.17	312	.00694	.00425	118	.00738
				Av.	.00700

Coil # 34.

E	I	Z	R	f	Mil Henries.
3.92	246	.0159	.00828	117	.0182
4.20	263	.0159	.00834	117	.0182
4.50	282	.0159	.00841	117	.0182
4.52	286	.0158	.00843	117	.0182
4.60	292	.0157	.00850	117	.0182
4.49	282	.0159	.00861	117	.0182
4.62	292	.0158	.00843	117	.0182
4.64	295	.0157	.00841	117	.0182
4.19	263	.0159	.00861	117	.0182
3.90	246	.0158	.00842	117	.0182
				Av.	.0182

DATA.

Coil # 35.

E	I	Z	R	f	Mil Henries.
4.56	118	.0387	.0160	118	.0477
5.27	138	.0385	.0157	118	.0477
5.77	151	.0382	.0158	118	.0470
6.14	159	.0386	.0149	118	.0477
6.15	162	.o376	.0159	118	.0462
5.77	153	.0377	.0156	118	.0462
5.52	146	.0378	.0156	118	.0464
5.75	152	.0378	.0156	118	.0464
6.00	158	.0378	.0156	118	.0464
5.92	155	.0381	.0155	118	.0470
				Av.	.0469

Coil # 36.

E	I	Z	R	f	Mil Henries.
5.25	68	.0780	.0288	117	.0984
6.00	76	.0788	.0287	116	.1000
6.82	88	.0776	.0287	117	.0983
7.12	92	.0773	.0286	117	.0980
7.66	99	.0773	.0288	117	.0980
8.18	105	.0778	.0287	117	.0983
8.50	110	.0773	.0288	117	.0980
				Av.	.0984

DATA.

Coil # 37.

E	I	Z	R	f	Mil Henries.
2.35	342	.00687	.00423	117	.00730
2.77	382	.00726	.00423	117	.00806
2/65	379	.00698	.00423	117	.00756
2.52	359	.00703	.00423	117	.00756
2.85	407	.00700	.00423	117	.00756
2.73	380	.00718	.00423	117	.00790
2.50	346	.00720	.00423	117	.00790
2.77	382	.00726	.00423	117	.00806
2.60	356	.00730	.00423	117	.00806
				Av.	.00777

Coil # 38.

3.40	226	.0151	.00839	118	.0169
4.20	278	.0151	.00839	118	.0169
4.80	331	.0145	.00839	117	.0161
5.25	356	.0145	.00839	117	.0161
5.60	379	.0148	.00839	117	.0166
5.30	356	.0149	.00839	117	.0166
4.90	329	.0149	.00839	118	.0166
5.35	358	.0149	.00839	118	.0166
5.65	380	.0149	.00839	118	.0166
5.07	338	.0150	.00838	118	.0167
				Av.	.0166

DATA.

Coil # 39.

E	I	Z	R	f	Mil Henries.
5.00	148	.0338	.0156	121	.0398
5.55	163	.0341	.0156	121	.0402
5.20	154	.0337	.0156	121	.0398
4.70	141	.0333	.0156	119	.0397
5.56	163	.0341	.0156	121	.0402
5.30	158	.0335	.0156	120	.0396
4.97	148	.0335	.0156	120	.0396
5.60	164	.0341	.0156	120	.0402
5.25	156	.0336	.0156	120	.0396
5.00	149	.0335	.0156	120	.0396
				Av.	.0398

Coil # 40.

8.55	135	.0634	.0287	111	.0810
9.05	143	.0634	.0286	111	.0810
9.55	152	.0628	.0288	111	.0800
10.54	168	.0624	.0287	110	.0800
9.60	153	.0628	.0287	111	.0800
9.12	145	.0628	.0287	111	.0800
8.50	134	.0634	.0287	111	.0810
				Av.	.0803

DATA.

C
Coil # 41.

E	I	Z	R	f	Mil Henries.
3.79	183	.0207	.00922	147	.0201
3.37	162	.0208	.00929	150	.0211
2.84	136	.0208	.00924	152	.0195
4.12	193	.0213	.00930	148	.0201
				Av.	.0204

Coil # 42.

4.29	169	.0253	.00952	135	.0277
3.75	148	.0253	.00952	134	.0279
3.15	115	.0274	.00942	137	.0299
2.70	104	.0259	.00945	137	.0281
4.96	190	.0261	.00949	138	.0280
				Av.	.0283

Coil # 43.

5.63	146	.0345	.0118	134	.0386
5.54	160	.0346	.0118	134	.0387
5.00	145	.0345	.0115	134	.0386
4.49	128	.0350	.0115	136	.0387
				Av.	.0387

DATA.

Coil # 44.

E	I	Z	R	f	Mil Henries.
2.82	139	.0203	.00966	134	.0215
3.39	174	.0194	.00962	135	.0210
3.84	211	.0182	.00950	133	.0180
4.15	227	.182	.00938	133	.0187
4.45	224	.o198	.00931	133	.0209
4.50	338	.0197	.00949	132	.0196
4.33	207	.0209	.00949	133	.0223
					Av. .0203

Coil # 45.

3.41	165	.0207	.0104	136	.0212
3.74	182	.0205	.0102	133	.0214
4.10	200	.0205	.0102	136	.0209
4.59	224	.0205	.0101	135	.0211
4.91	239	.02o5	.0101	135	.0211
4.76	229	.0208	.0101	135	.0214
4.51	213	.0212	.0101	135	.0219
4.15	195	.0213	.0100	136	.0219
3.82	179	.0215	.0101	136	.0219
3.64	170	.0214	.0101	137	.0219
					Av. .0215

DATA.

Coil # 46.

E	I	Z	R	f	Mil Henries.
4.76	240	.0198	.0103	132	.0205
4.34	214	.0203	.0104	134	.0208
3.86	189	.0204	.0102	135	.0208
4.07	201	.0202	.0101	133	.0209
4.35	213	.0204	.0101	134	.0210
4.71	229	.0206	.0101	133	.0213
5.11	246	.0208	.0098	133	.0216
4.53	215	.0211	.0098	133	.0220
3.81	188	.0303	.0102	135	.0206
3.59	170	.0211	.0102	135	.0218
					Av. .0211

Coil #47.

3.10	152	.0204	.0091	139	.0210
3.49	168	.0208	.0106	139	.0214
3.68	176	.0209	.0091	139	.0216
3.95	189	.0209	.0091	138	.0217
4.18	199	.0210	.0090	138	.0219
4.00	189	.0212	.0090	138	.0221
3.88	183	.0212	.0090	137	.0223
3.73	174	.0214	.0090	138	.0224
3.85	183	.0210	.0090	138	.0219
4.o#	191	.0211	.0090	139	.0219
					Av. .0218

DATA.

Coil # 48.

E	I	Z	R	f	Mil Henries.
3.57	171	.0209	.0100	137	.0215
3.79	181	.0209	.0110	136	.0216
3.95	189	.0209	.0099	134	.0220
4.20	196	.0214	.0097	136	.0223
4.17	194	.0215	.0097	137	.0223
4.03	186	.0216	.0096	137	.0224
3.96	183	.0216	.0095	136	.0226
3.79	176	.0213	.0096	136	.0225
3.91	183	.0214	.0097	136	.0223
3.80	176	.0216	.0098	136	.0222
				Av.	.0222

Coil # 49.

3.53	128	.0276	.0098	139	.0296
3.82	140	.0273	.0097	138	.0294
4.03	146	.0276	.0096	138	.0298
4.48	164	.0273	.0097	137	.0297
4.61	168	.0274	.0096	138	.0296
4.44	161	.0276	.0097	137	.0300
4.52	163	.0276	.0095	138	.0300
4.34	156	.0278	.0095	138	.0300
4.58	164	.0279	.0096	138	.0300
4.52	163	.0277	.0095	138	.0300
				Av.	.0298

DATA.

Coil # 50.

E	I	Z	R	f	Mil Henries.
3.92	145	.0270	.0110	139	.0283
4.15	146	.0284	.0109	137	.0305
4.39	155	.0283	.0109	137	.0304
4.58	159	.0288	.0108	137	.0310
4.68	163	.0287	.0109	137	.0308
4.63	160	.0289	.0109	137	.0311
4.54	154	.0295	.0107	136	.0320
4.37	148	.0295	.0106	137	.0319
4.54	154	.0295	.0107	138	.0316
4.64	158	.0294	.0108	137	.0317
					Av. .0309

DIAMETER OF MAGNET WIRE.

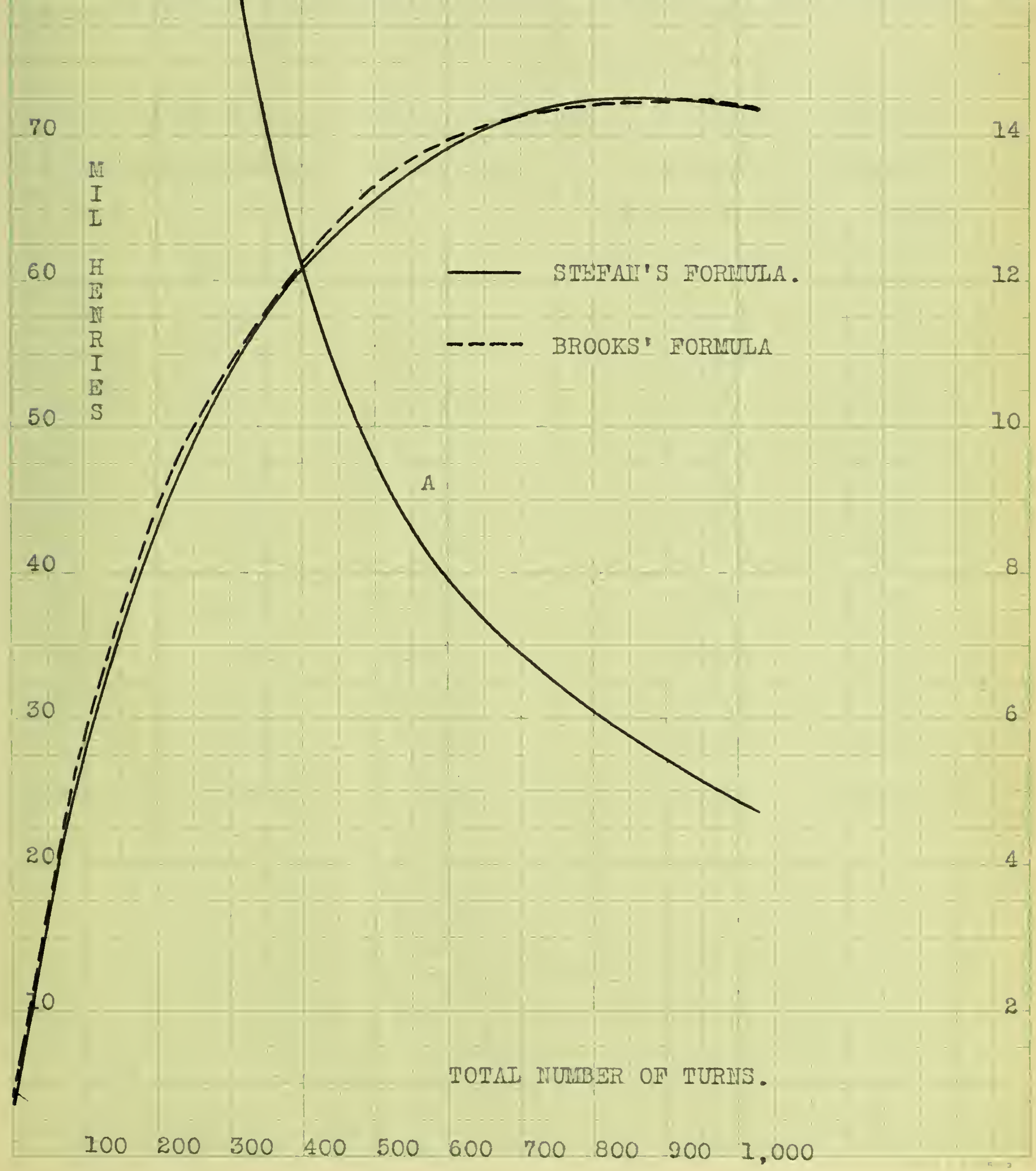
(Roebling's)

B.& S.G.	Diameter drawn in Cm.	Outside diameter in Centimeters	
		Single Cotton	Double Cotton
0	.826	.856	.871
1	.734	.765	.779
2	.655	.686	.701
3	.582	.612	.627
4	.518	.549	.564
5	.462	.493	.508
6	.411	.437	.452
7	.366	.391	.406
8	.325	.348	.361
9	.290	.310	.320
10	.259	.274	.284
11	.231	.246	.257
12	.206	.220	.231
13	.183	.198	.206
14	.163	.178	.185
15	.145	.160	.168
16	.130	.145	.152
17	.114	.130	.137
18	.101	.117	.124
19	.091	.107	.114

FIGURE I.

CURVE SHOWING THE VARIATION OF INDUCTANCE WITH THE RADIUS AND NUMBER OF TURNS, THE CROSS SECTION OF THE COIL WAS SQUARE, THAT IS, l=t. 30480 CENTIMETERS OR 1,000 FEET OF DOUBLE COTTON COVERED COPPER WIRE WAS USED IN EACH CASE.

THE ORDINATES TO CURVE A GIVE THE RADIUS IN CENTIMETERS FOR THE NUMBER OF TURNS INDICATED BY THE ABSCISSA.



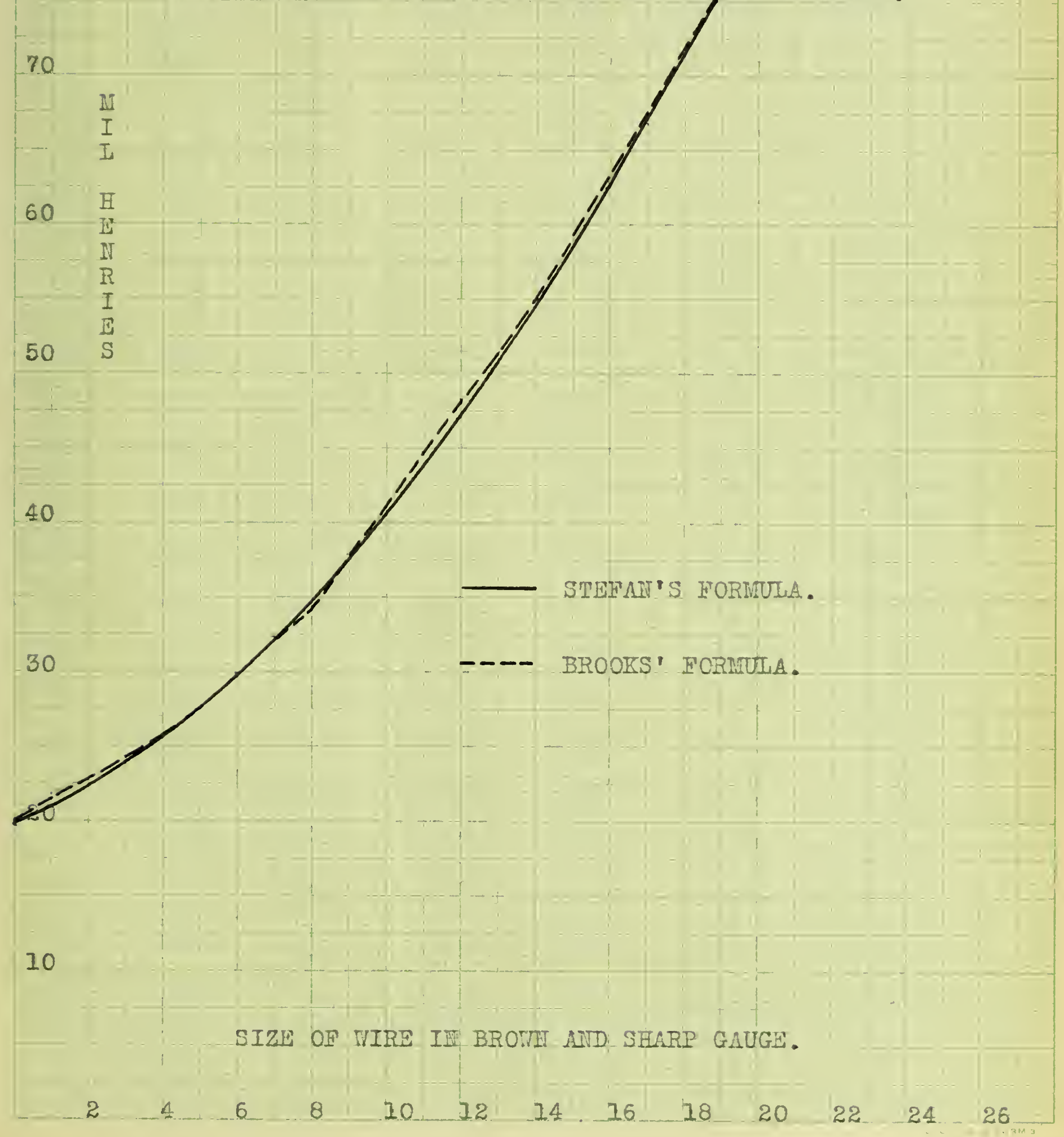
DATA FOR FIGURE I.

Curve showing the variation of inductance with the radius and numbers of turns, the cross section of the coil was square, that is, l=t, 30480 centimeters or 1,000 feet of double cotton covered copper wire was used in each case.

Number of turns	Mean Radius Centimeters	Length = Thickness Centimeters	Stefan's Formula Mil Henries	Brooks' Formula Mil Henries
1	4851.00	0.12	0.417	0.42
100	48.51	1.24	27.740	28.20
225	21.56	1.87	45.650	47.20
400	12.13	2.49	60.700	60.70
484	10.02	2.74	64.850	66.10
676	7.17	3.23	70.620	71.00
729	6.65	3.36	71.490	72.00
784	6.18	3.48	71.860	72.10
841	5.77	3.61	72.420	72.80
900	5.39	3.73	72.480	72.50
961	5.05	3.86	72.290	72.40
1024	4.74	3.98	71.940	71.80

FIGURE II.

CURVE SHOWING THE VARIATION OF INDUCTANCE WITH THE SIZE OF WIRE. 30480 CENTIMETERS OR 1,000 FEET OF DOUBLE COTTON COVERED WIRE OF THE SIZES INDICATED BY THE ABSCISSA WAS WOUND FOR MAXIMUM INDUCTANCE.



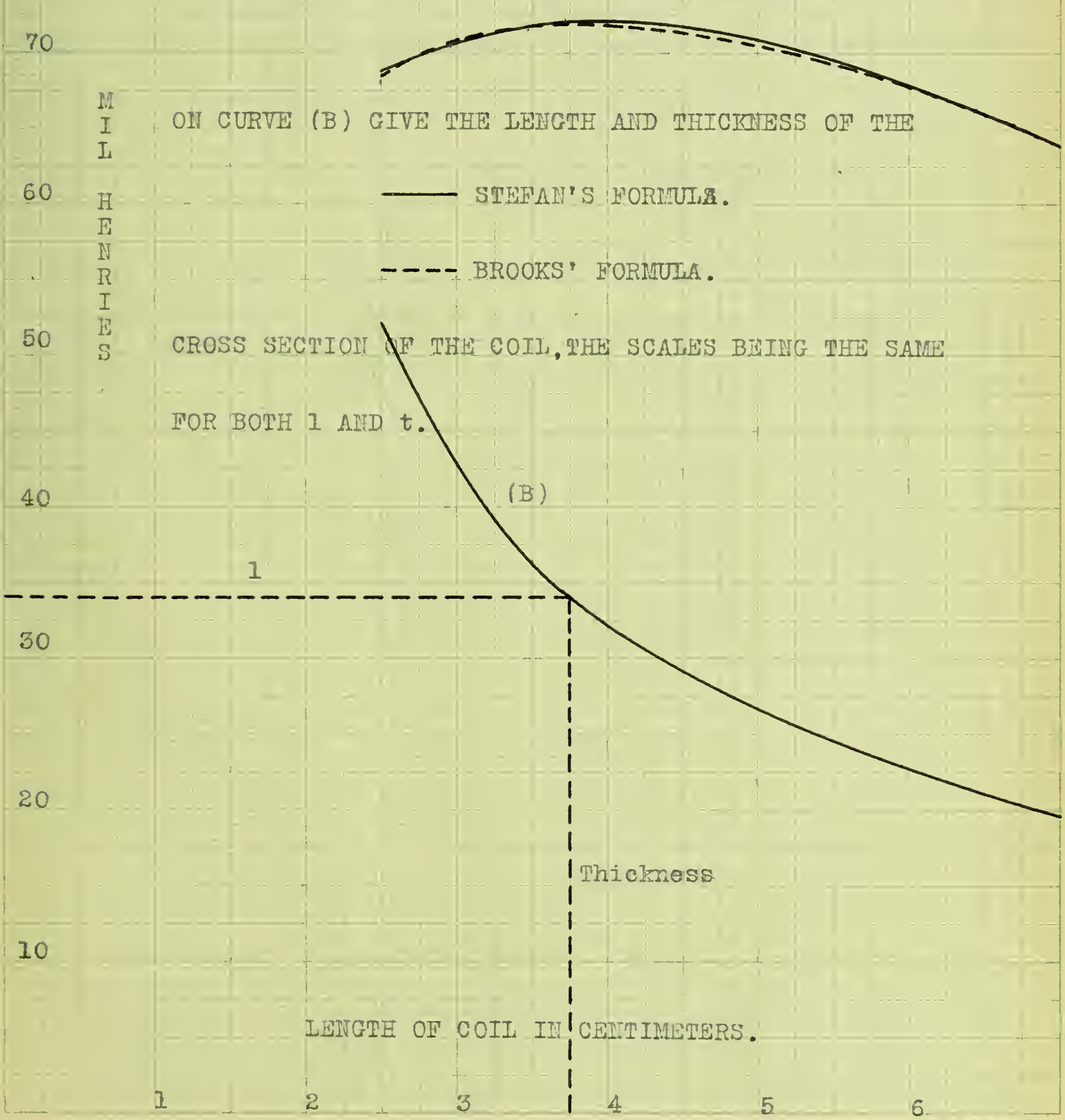
DATA FOR FIGURE II.

30480 centimeters = 1,000 feet of double cotton covered wire of the numbers shown in table was wound for maximum inductance in the most practical manner. The cross section was square.

# of wire	Total # of turns	Mean radius of coil	Radius of core cm.	Length of coil cm.	Brooks' formula Mil Henries	Stefan's formula Mil Henries
0	225	21.56	15.03	13.07	19.8	19.77
1	225	21.56	15.71	11.70	21.8	21.67
2	256	18.95	13.34	11.22	22.7	22.69
3	256	18.95	13.93	10.04	24.3	24.38
4	289	16.79	11.99	9.59	26.0	26.04
5	289	16.79	12.47	8.64	27.1	28.00
6	324	14.97	10.90	8.14	29.7	29.59
7	361	13.44	9.58	7.72	32.4	32.80
8	361	13.44	10.01	6.85	34.4	35.24
9	400	12.13	8.93	6.40	37.8	38.22
10	441	11.00	8.01	5.97	41.0	41.40
11	484	10.02	7.20	5.64	44.7	43.89
12	484	10.02	7.48	5.09	48.3	48.96
13	529	9.17	6.80	4.73	51.4	51.03
14	576	8.42	6.20	4.45	54.9	55.04
15	625	7.76	5.67	4.19	56.8	57.53
16	676	7.18	5.20	3.96	63.0	62.92
17	729	6.65	4.80	3.70	67.7	68.55
18	784	6.19	4.45	3.49	71.6	71.65
19	784	6.19	4.59	3.20	76.1	75.71

FIGURE III.

CURVE SHOWING THE VARIATION OF INDUCTANCE WITH THE FACTORS l AND t, THE FOLLOWING FACTORS WERE CONSTANT, lt, LENGTH OF WIRE (30480 CM. = 1,000 FT.), MEAN RADIUS (5.775 CM.). THE COORDINATES OF CORRESPONDING POINTS ON CURVE (B) GIVE THE LENGTH AND THICKNESS OF THE CROSS SECTION OF THE COIL, THE SCALES BEING THE SAME FOR BOTH l AND t.



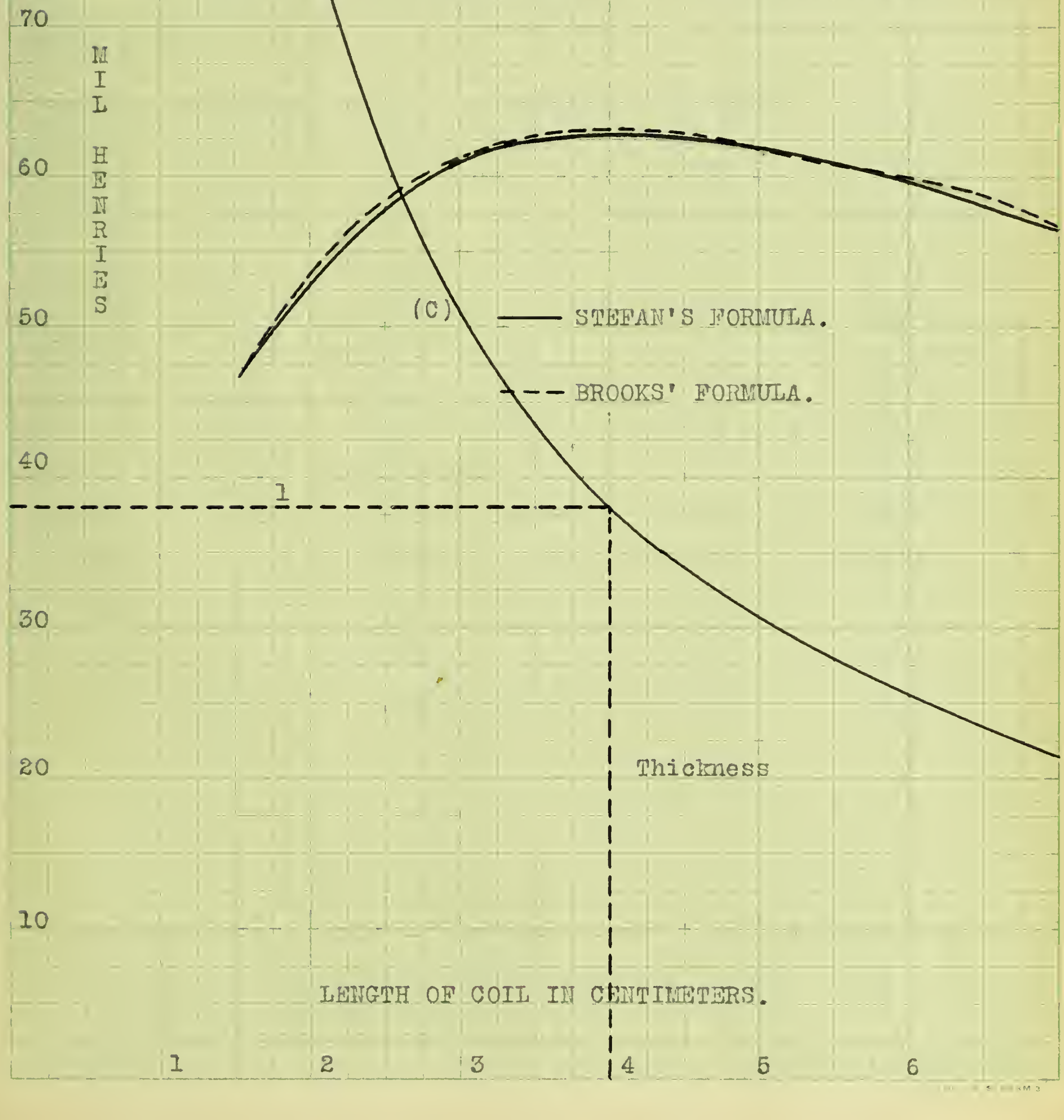
DATA FOR FIGURE III.

Variation of inductance with the factors l and t, the following factors were constant, lt, length of wire (30480 cm. = 1,000 ft.). The mean radius used was 5.775 centimeters. The mean radius was selected so as to give approximately the maximum inductance from the given wire. The curve shows that, for the radius selected, the maximum inductance is obtained when the length is slightly greater than the thickness. Number of turns equal 840.

Length in turns	Thickness in turns	Length in centimeter	Thickness in cm.	Stefan's formula Mil Henries	Brooks' formula Mil Henries
30	28	3.73	3.48	72.10	72.0
28	30	3.48	3.73	71.90	71.5
35	24	4.36	2.98	71.76	71.5
24	35	2.98	4.36	70.60	70.5
42	20	5.23	2.49	69.69	69.5
20	42	2.49	5.23	68.71	68.0
84	10	10.45	1.22	53.96	53.8

FIGURE IV.

CURVE SHOWING THE VARIATION OF INDUCTANCE WITH THE FACTORS l AND t, THE FOLLOWING FACTORS WERE CONSTANT, lt, LENGTH OF WIRE (30480 CM. = 1,000FT.), MEAN RADIUS (7.35 CM.). THE WIRE USED WAS # 16 DOUBLE COTTON COVERED COPPER WIRE. THE COORDINATES OF THE CORRESPONDING POINTS ON CURVE (C) GIVE THE LENGTH AND THICKNESS OF THE CROSS SECTION OF THE COIL, THE SCALE BEING THE SAME FOR BOTH l AND t.



DATA FOR FIGURE IV.

Six hundred and sixty turns of number 16 double cotton covered copper wire was wound to a mean radius of 7.35 centimeters, the length and thickness of the cross section were varied while their product was kept constant. The length of the wire taken was 30480 centimeters or 1,000 feet. The mean radius was selected so as to give approximately the maximum inductance obtainable from the given wire.

Length in turns	Thickness in turns	Length in cm.	Thickness in cm.	Stefan's formula Mil Henries	Brooks' formula Mil Henries
33	20	5.03	3.05	61.8	61.5
20	33	3.05	5.03	60.8	61.3
30	22	4.57	3.35	62.6	62.8
22	30	3.35	4.57	62.2	62.0
44	15	6.70	2.29	58.0	59.0
15	44	2.29	6.70	56.4	57.1
55	12	8.30	1.83	52.2	52.3
12	55	1.83	8.30	50.3	51.5
66	10	10.06	1.54	50.0	50.2
10	66	1.54	10.06	46.7	47.2

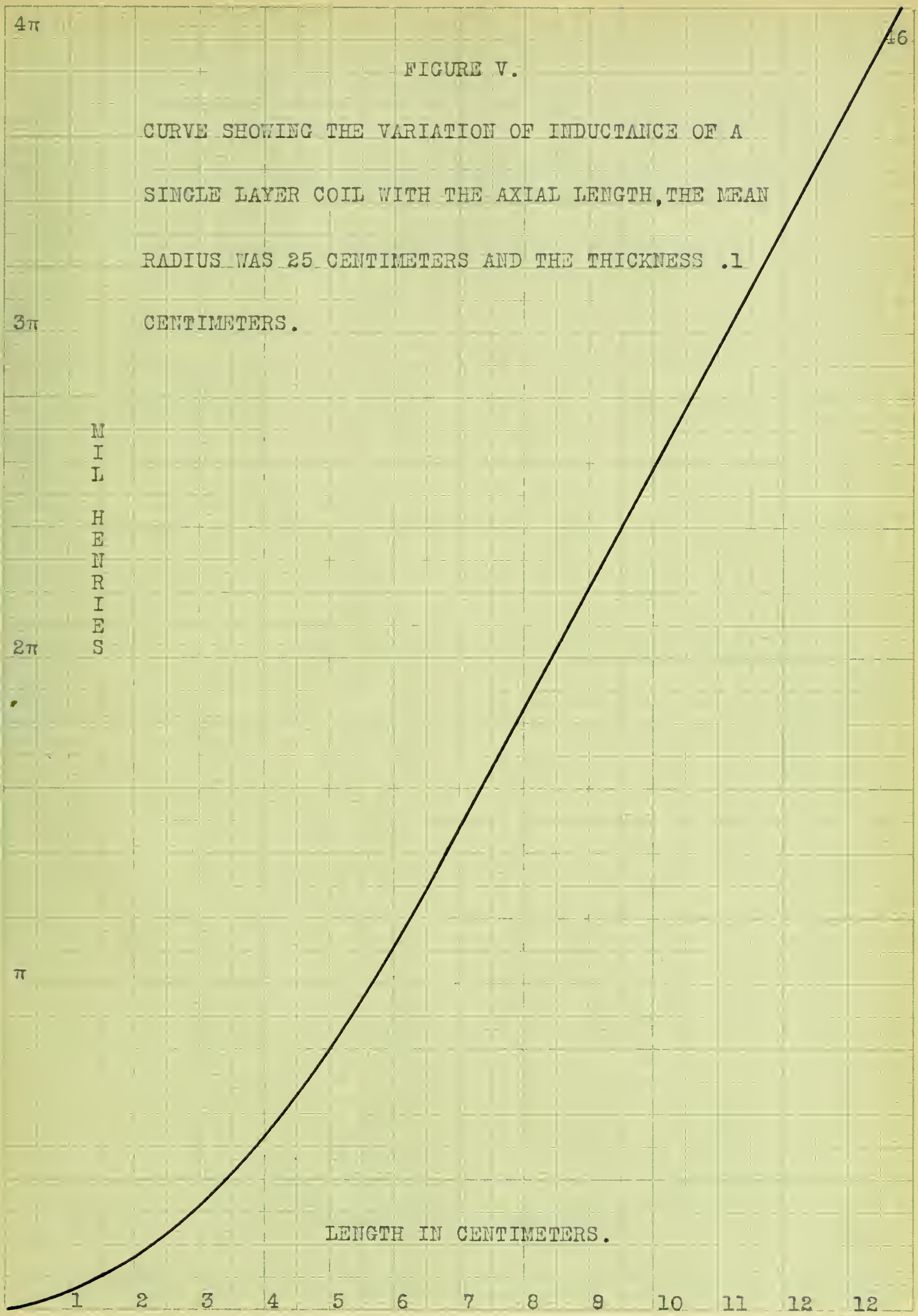
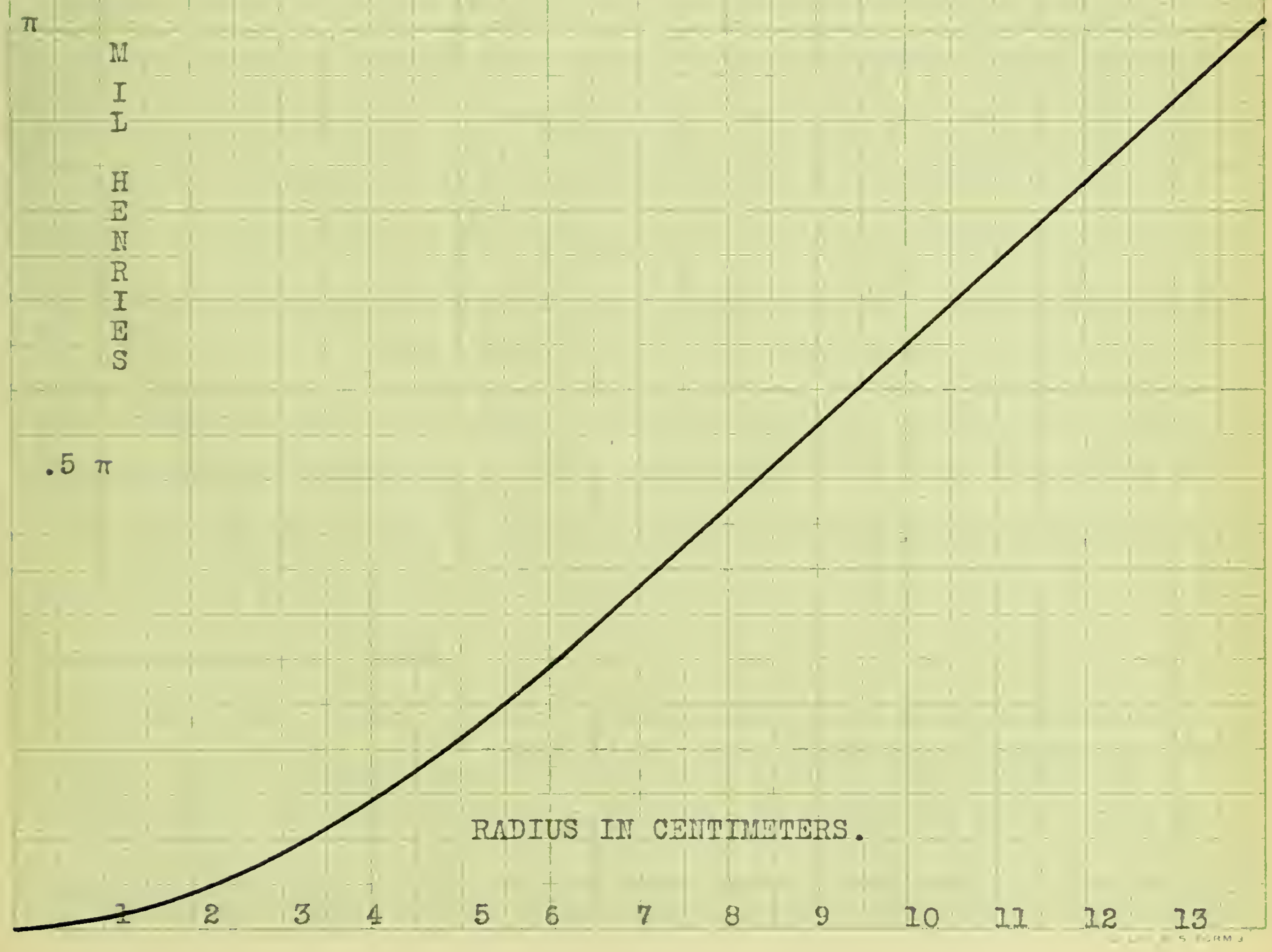


FIGURE V.

CURVE SHOWING THE VARIATION OF INDUCTANCE OF A SINGLE LAYER COIL WITH THE AXIAL LENGTH, THE MEAN RADIUS WAS 25 CENTIMETERS AND THE THICKNESS .1 CENTIMETERS.

FIGURE VI.

CURVE SHOWING THE VARIATION OF INDUCTANCE OF A SINGLE LAYER COIL WITH THE RADIUS, THE LENGTH WAS 10 CENTIMETERS AND THE THICKNESS .1 CENTIMETER.



REACTANCE COILS.

The formula for inductance may be reduced to the following form,

$$L = C^2/l$$

as has previously been shown, and from this form several interesting conclusions may be drawn, viz; when the space factor is assumed to be a constant proportion (not quite true in the case of magnet wire, especially in the smaller sizes) the weight of wire in a coil of maximum inductance, having the same relative proportions, will be in the proportion to the third power of any dimension, such as D. The denominator is then proportional to the cube root of the weight of the wire. The numerator for a given size of wire is proportional to the length of the wire which in turn is proportional to the weight. If the size of the conductor varies, the weight varies directly as the length, and as the square of the diameter. The reactance and the resistance of a certain piece of wire such as a 1,000 feet of number 10 being known the reactance and the resistance of any other size can readily be approximated. For instance, with number 13 wire, which has half the area of number 10, twice the length will be required to fill the given spool, and since the reactance has its numerator made four times as great and the denominator remains unchanged, the reactance is four times as great. The resistance is also four times as great, and the ratio of X/R remains constant for any size of wire, assuming the space factor of the insulation to be the same in both cases.

If the size of the spool is increased the relative proportions of the coil remaining constant so as to obtain the maximum reactance, and the spool filled with greater length of wire of the same gauge number, the numerator increases as the square of the

length,the denominator increases as the cube root of the length, and the reactance increases by the 5/3 power of the length. But, since the resistance varies directly with the length,the ratio , X/R varies as the 2/3 power of the length. For a given gauge of wire,the reactance gains faster than the resistance with increasing length,and,the relation of X/R being definite for a spool of given dimensions,it is easy to determine (for a given frequency) the absolute dimensions of a spool required to prodice any desired inductance factor in a coil.

MAXIMUM INDUCTANCE.

MAXWELL'S APPROXIMATE FORMULA.

$$L = 4\pi a n^2(\log(8a/R) - 2) \tag{1}$$

Let l' equal the total length of wire in coil, then

$$l' = 2\pi a n \tag{2}$$

$$l' = 2\pi C R^2 a \tag{3}$$

Where C is a constant, R = G. M. D., n the number of turns of wire in the coil, and a the mean radius.

Differentiating (2) and (3) with respect to (a) we have

$$dn/da = -n/a \text{ and } dR/da = -R/2a$$

Differentiating (1) with respect to (a) we have

$$dL/da = 4\pi(2\,n\,a(\log 8a/R\ -2)dn/da + n^2(\log 8a/R\ -2) + n^2 - n^2 a/R\ (dR/da))$$

Substituting the value of dn/da and dR/da we have

$$dL/da = 4\pi\ (-2n^2(\log 8a/R - 2) + n^2(\log 8a/R - 2) + n^2 + n^2/2)$$

Equating dL/da to zero we have

$$dL/da = 0 = 4\pi n^2(7/2\ - \log 8a/R), \text{then}$$

$$\log(8a/R) = 7/2, \text{or}$$

$$\log R = \log a\ - 7/2$$

From the Bureau of Standards Bulletins

$$\log R = \log r\ - 1/4$$

for circular cross sections, where r is the radius of the cross section, then

$$\log r\ - 1/4 = \log 8a\ - 7/2$$

$$\log 8a/r = 13/4$$

$$a = (1/8)e^{3.25}r = 3.224\ r$$

But for square cross sections, that is, where l = t,

$$R = 0.2235\ (l+t) = 0.447\ l \text{ (letter l)}$$

$$\log 8a - 7/2 = \log 0.447\, l \text{, or}$$

$$\log 8a - \log 0.447\, l = 7/2$$

$$\log (8a/0.447\, l) = 7/2 \text{, then}$$

$$8a/0.447\, l = e^{7/2}$$

$$a = (0.447\, l\, e^{7/2}$$

$$= 1.85\, l \text{ (letter l).}$$

GENERAL CASE.

When it is desired to get the maximum inductance out of a wire of given length and diameter the following offers an easy means of determining the radius and axial length of coil.

Let l' be the length of the conductor,

n the number of turns,

a the mean radius, and

d the diameter of the wire.

$$2\pi a n = l' \qquad (1)$$

$$a n = l'/2\pi \qquad (2)$$

But a equal $1.85\, l$ for square cross sections then

$$l n = l'/(2\pi \times 1.85)$$

But $l = t = \sqrt{n} \times d$ for square cross sections

$$n^{3/2} = l'/(2\pi \times 1.85 \times d)$$

Substituting for l' and d for any particular case we can find the value of n. Then from (1) and (2° we can determine the value of a, l, and t.

DESCRIPTION OF COILS.

All dimensions are in centimeters.

Coil	Layers	Turns	Length	Thickness	Mean Radius	Length of Conductor
#1	28	784	3.2	3.20	6.19	30480
2	10	330	3.0	0.79	2.90	6003
3	14	462	3.0	1.15	3.08	8926
4	18	594	3.0	1.59	3.25	12111
5	20	660	3.0	1.65	3.28	13511
6	1	1	0.2	0.20	25.00	157
7	1	1	0.1	0.10	25.00	157
8	1	1	1.0	1.00	25.00	157
9	1	2	0.2	0.10	99.85	627
10	2	4	0.2	0.20	99.90	2507
11	1	4	0.4	0.10	100.00	2513
12	1	10	1.0	0.10	25.00	1571
13	1	20	2.0	0.10	25.00	3142
14	4	16	0.4	0.40	100.00	10053
15	1	50	5.0	0.10	20.00	6283
16	10	100	1.0	1.00	4.00	1257
17	1	1	2.0	2.00	10.00	63
18	1	1	1.0	1.00	25.00	157
19	20	400	1.0	1.00	10.00	25133
20	1	1	.65	.65	100.00	628
21	1	2	1.75	.65	100.00	1257
22	1	3	2.85	.65	100.00	1885
23	1	4	3.96	.65	100.00	2514
24	1	8	8.35	.65	100.00	5028
25	1	1	0.65	.65	100.00	628

DESCRIPTION OF COILS.

All dimensions are in centimeters.

Coil	Layers	Turns	Length	Thickness	Mean Radius	Length of Conductor.
26	1	2	2.5	.65	100	1257
27	1	4	7.5	.65	100	2514
28	1	8	17.5	.65	100	5028
29	1	1	0.65	.65	100	628
30	1	2	5.0	.65	100	1257
31	1	4	15.0	.65	100	2514
32	1	8	35.0	.65	100	5028
33	1	1	0.65	.65	100	628
34	1	2	7.5	.65	100	1257
35	1	4	22.5	.65	100	2514
36	1	8	52.5	.65	100	5028
37	1	1	0.65	.65	100	628
38	1	2	10.0	.65	100	1257
39	1	4	30.0	.65	100	2514
40	1	8	70.0	.65	100	5028
41	1	10	23.38	.52	15.24	957.5
42	1	10	11.95	.52	15.24	957.5
43	1	10	5.22	.52	15.24	957.5
44	1	10	23.38	.52	15.24	957.5
45	1	10	23.38	.52	15.24	957.5
46	1	10	23.38	.52	15.24	957.5
47	1	10	23.38	.52	15.24	957.5
48	1	10	23.38	.52	15.24	957.5
49	1	10	11.95	.52	15.24	957.5
50	1	10	5.22	.52	15.24	957.5

COMPARISON.

Coil	By Experiment	Prof. Brooks' Formula	Error
1	75.754	76.10	+ 0.4 %
2	5.543	5.578	+ 0.64 %
4	18.476	18.989	+ 2.77 %
5	24.110	23.511	- 2.49 %
	BUREAU OF STANDARDS		
6 (round)	.02058	.00204	-0.88 %
7 (square)	.00202	.00204	+1.22 %
8	.00041	.000416	+1.39 %
9	.0374	.0372	-0.53 %
10	.1435	.1410	-1.75 %
11	.1394	.1397	+0.31 %
12	.1490	.1506	+0.89 %
13	.5150	.5220	+1.64 %
14	2.070	2.050	-0.97 %
15	1.860	1.900	+1.93 %
16	1.147	1.176	+2.53 %
17	.000331	.000326	-1.51 %
18	.00133	.00131	-1.50 %
19	64.158	65.80000	+2.55 %
	Ordinary formulae do not apply to this form of coil.		
20	.00684	.00742	+ 5.55 %
21	.0231	.00275	+18.6 %
22	.0447	.00541	+20.8 %
23	.0669	.0095	+23.6 %

Coil	By Experiment	Prof. Brooks' Formula	Error
24	.146	.329	+125.0 %
25	.00809	.00742	– 8.3 %
26	.0214	.0325	+ 52.0 %
27	.0589	.1001	+ 70.0 %
28	.1280	.3080	+140.0 %
29	.00716	.00742	+ 3.6 %
30	.0202	.0247	+ 22.0 %
31	.0536	.1070	+100.0 %
32	.1160	.3240	+102.0 %
33	.0070	.00742	+ 6.0 %
34	.0182	.0252	+ 38.% %
35	.0469	.0696	+ 48.4 %
36	.0948	.1960	+106.0 %
37	.0077	.0074	+ 4.5 %
38	.0166	.0232	+ 39.6 %
39	.0398	.0622	+ 57.0 %
40	.0803	.1930	+139.0 %
41	.0204	.0245	+ 20.0 %
42	.0283	.0361	+ 27.6 %
43	.0387	.0504	+ 30.2 %
44	.0203	.0245	+ 20.2 %
45	.0215	.0245	+ 14.0 %
46	.0211	.0245	+ 16.1 %
47	.0218	.0245	+ 12.4 %
48	.0222	.0245	+ 10.3 %
49	00298	.0245	+ 28.3 %
50	.0309	.0504	+ 65.0 %

CONCLUSION.

The final comparison,found on page fifty four,of the formula being investigated consists in checking the formula against the inductance of coils of known value and values obtained experimentally, by methods previously explained,while those from six to nineteen inclusive were obtained from the bulletins of the Bureau of Standards. By refering to the description of the coils it will be seen that coils of from one to seven hundred and eighty four turns,varying in proportion from circles to square and rectangular cross sections were investigated and in every case the formula was well within the limits of error for all practical work.

These considerations lead to the conclusions that the formula is applicable to all practical problems that can be solved by means of the more accurate formulae.

The data on coils from twenty to fifty inclusive was taken with the hope that a pitch factor could be found so as to make the formula applicable to helical coils of considerable pitch but lack of time prevented carrying out this idea. The difference between the values obtained experimentally and those obtained by use of the formula do not in any way detract from its value since other formulae,more exact for the ordinary shaped coil,gave results showing greater error.